Improved reservoir level assessment through the mathematical modelling of weir crest coefficients

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SYNOPSIS. An accurate assessment of reservoir levels is a key part of any dam safety review. Assessing the catchment hydrology will yield incoming floods but their accurate routing to assess associated reservoir levels will also require an accurate assessment of outlet work discharge characteristics. In the case of simple overspill weirs the discharge coefficients are all too often guessed or estimated as constant values, whereas in fact they are more likely to vary with head.

Recent statutory inspections at Loyne and Cluanie dams revealed that weir discharge coefficients of 1.57, 1.63, 1.71 and 2.00 had all been used at different times in the past, by different engineers for essentially the same structures. For the inspection in 2005 the free flow surfaces over both weirs were simulated using computational fluid dynamics (CFD). This enabled the weir discharge coefficients to be assessed for a range of flows. The use of these for subsequent flood routing reduced reservoir levels at both dams.

The paper describes this work and gives recommendations for more simplified, assessments at other dams in the future.

INTRODUCTION

The Loyne and Cluanie dams were completed in 1960. Both are concrete gravity structures with similar design details and profiles. The upstream and downstream faces of both dams were formed using pre-cast units to retain an internal hearting concrete. The upstream faces of the dams are vertical and the downstream faces slope at 0.7 on 1.0. The central sections of both dams have long, un-gated spillways featuring profiled crests with flat upstream extensions. The spillway crest length at Loyne dam is 68.58m and the effective length at Cluanie, 129.39m, with bridge pier widths subtracted. Both dams are owned and operated by Scottish & Southern Energy (SSE).

Stability analyses, carried out in recent years for both dams, as part of SSE's portfolio seismic and PMF stability reviews, indicated that safety factors are marginal and sensitive to the maximum water level reached during floods. In view of this a number of probable maximum flood (PMF) variants have been mathematically routed through the reservoirs in order to establish a probable maximum reservoir level. Surprisingly, however, there has been little refinement, or indeed consensus, on the associated discharge characteristics to be used for the overspill crests. The original designers suggested weir discharge coefficients of only 1.566 for Loyne and 1.626 for Cluanie. These are surprisingly low, in fact lower than a basic broad crested weir coefficient value of 1.71. In view of this, the 1995 statutory inspection adopted a value of 1.71 in checking freeboard adequacy. Even this, however, is lower than one might reasonably expect from a hydraulically profiled crest. Subsequent reviews of reservoir level for stability calculations adopted a discharge coefficient of 2.00 for both dams. The spillway crest of Loyne dam is shown on Figure 1.

Figure 1. The overspill crest of Loyne dam

Clearly the flood levels achieved will be directly dependent on the weir coefficient and it is surprising that such a wide range of values has been used for these dams over the years. In view of this, and with the agreement of Scottish & Southern Energy, the most recent statutory inspection also featured a more accurate derivation of the discharge characteristics of these crests using computational fluid dynamics, (CFD) modelling techniques. This was used to simulate the spill over the dams for a range of flow rates. The paper describes how this was done, the results obtained and how these affected derived reservoir levels in both cases.

MODELLING METHOD

The concept of CFD modelling

The CFD models were assembled using CFX5.7 which is a CFD commercial code widely used in the aerospace, nuclear energy, automotive and marine industries. CFD involves the numerical modelling of fluid motion based on the application of basic physical principles. The first step in building a CFD model is to set up a three-dimensional mesh which splits the fluid into a large number of small elements. The behaviour of these elements is then predicted using the Navier-Stokes set of simultaneous differential equations describing fluid flow. There are normally five sets of equations covering:

- Conservation of mass
- Conservation of momentum (three equations, one for each dimension)
- Conservation of energy

In order to simulate a free surface, there is the additional complication of solving for two different fluids (air and water) and tracking the interface between the air and water. CFX5.7 does this using a model based on the "Volume of Fluid" method which calculates the volume fraction of each fluid in each element for each time step. A limitation of free surface modelling with CFD is that it is computationally intensive and models typically take several times longer to run than an enclosed (single phase) model with a similar mesh size.

The model domain or geometry

The models were initially assembled in imperial units. This was done for convenience as the available drawings were in feet and inches. However all results were then converted to metric units for further use. The model domain (geometry) is shown in Figure 2. There are a number of notable features:

- The model is 2 dimensional (one element wide). This reduces the computational time.
- The actual reservoirs are several kilometres long and the dams 21m and 40.5m high in the cases of Loyne and Cluanie respectively. To prevent excessive computational run time, the effective reservoir reaches were modelled as 50 ft upstream of the dams and with depths of 20 ft below the crests. When truncating a model, it is important to ensure that such dimensions do not overly influence the solution.
- The downstream profiles of the dams were modelled in full to just beyond the point at which the profiles transferred into constant batters. A curve was then included to return the flow to the horizontal. This curve was not a real feature of the dam, but was included to simulate supercritical flow at the model outlet and hence simplify the set-up of boundary conditions.
- Inlet conditions were simulated as an expanding taper with a separate air inlet above. This was done to simplify the set-up of boundary conditions as keeping the inlet fully submerged also keeps the inflow as 100% water. The taper allows the water depth to gradually increase to the free surface depth as determined by the model.

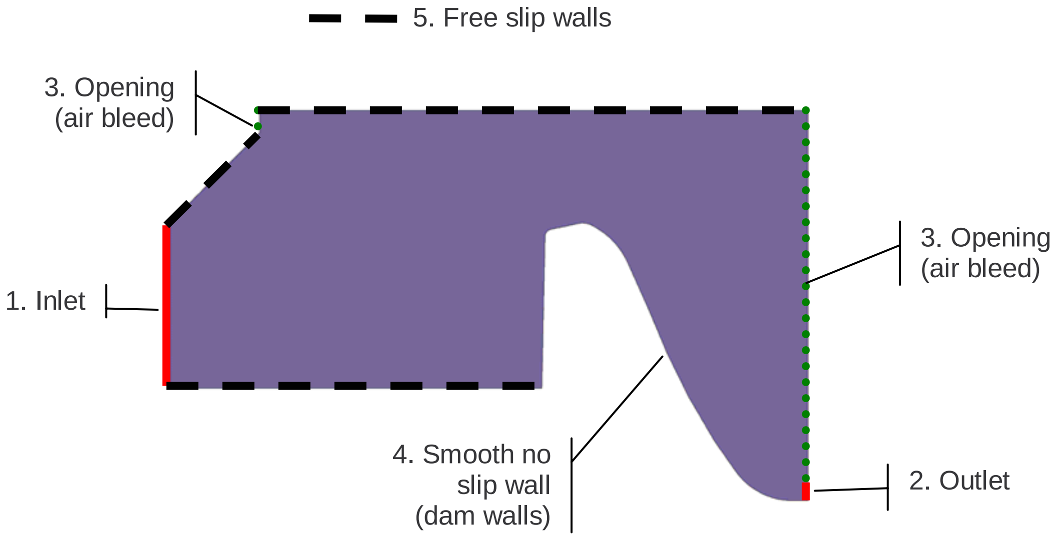


Figure 2. Model domain and boundary conditions

The mesh

The simulations were first run on a coarse mesh to obtain an initial approximate solution. A second set of simulations was then run using a finer mesh with the results from the initial runs used as the starting conditions. This two stage approach reduces the overall computational time and has the added benefit that preliminary results are quickly available. Key details of the meshes are summarised in table 1.

Table 1. Mesh Details

Feature	Coarse mesh	Fine mesh
Predominant mesh type	Unstructured prismatic wedge mesh	Unstructured prismatic wedge mesh
hexahedral inflation off the dam walls	5 layers to depth of 1 foot	5 layers to depth of 1 foot
Global mesh size	1 foot	0.5 foot
Refined mesh near free surface	0.3 foot	0.1 foot
Angular resolution	12	12
Approx. number of elements	20,000	90,000

An automatically generated unstructured mesh was used due to the speed with which it could be set up, although it would have been possible to build a more efficient structured mesh if minimising model run time had been a priority.

A partial mesh for one of the flow rates tested is shown in Figure 3. The meshes were set up with smaller mesh spacing in the vicinity of the anticipated water surface. This enables more accurate calculation of the free surface profile. For this reason a different mesh was set up for each flow rate simulated.

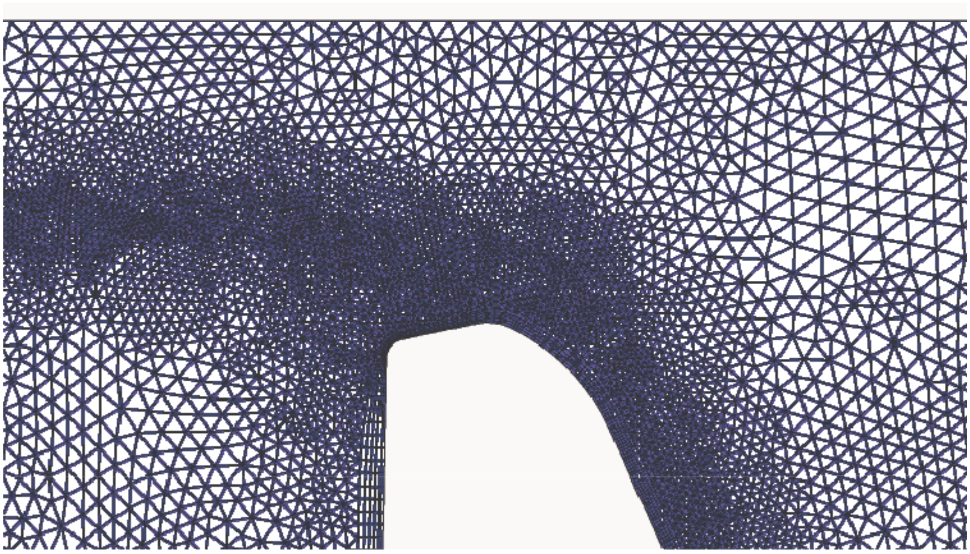
The Boundary Conditions

The following boundary conditions were specified (see Figure 2 for location):

1. Inlet: Mass flow of water
2. Outlet: Hydrostatic pressure profile – set at estimated height of downstream water level. The height used is not critical to the model performance provided the predicted flow remains supercritical.

3. Openings: Inflow or outflow unspecified. Water and air are free to flow out of the domain. All flow into the domain is specified as 100% air. These openings are effectively air bleeds.
4. Free slip walls (no restrictions to flow).
5. No slip smooth wall (velocities forced to be zero at the walls but without any assigned, wall roughness value).

Coarse mesh



Fine mesh

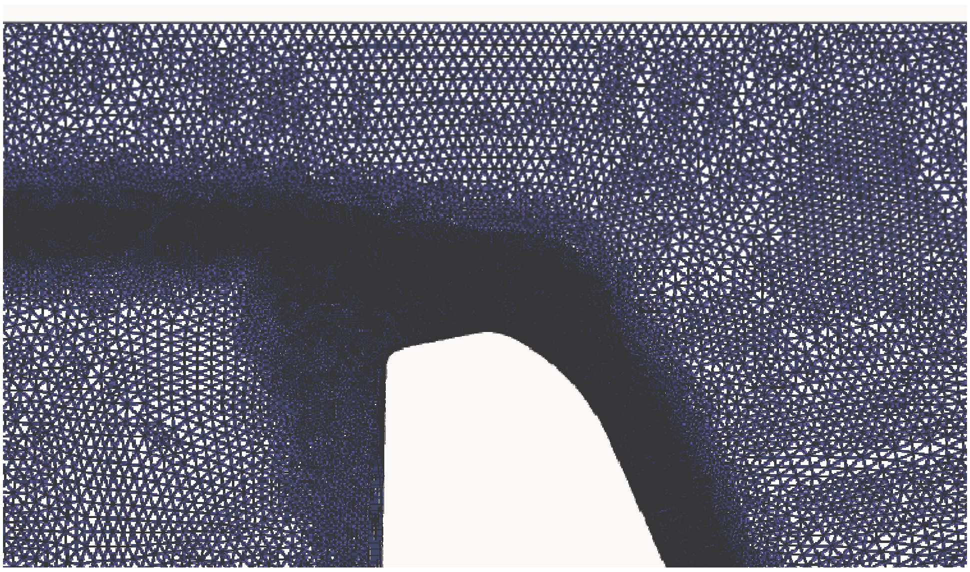


Figure 3. Meshes

Set-up and run time details

The simulation was set up in a fairly routine way for free surface modelling of water flows, the relevant factors are included here for reference but are not discussed in detail:

- Steady state
- k-epsilon turbulence model
- Homogeneous free surface model (based on the VOF method).
- Convergence criteria 1×10^{-4} RMS error

Free surface modelling is often constrained by computing power. To accelerate the convergence a high physical timescale was used at the start of the simulation. This value was set by trial and error, but could be up to 100 times higher than the default timescale. To help determine when the free surface had stabilised, the pressure at several locations upstream of the dam was monitored throughout the simulation.

The coarse mesh models typically took 2 to 4 hours to run, the fine mesh models took up to 24h to run on a twin 2.4Ghz processor computer. Interestingly it was found that the additional mesh refinement gave less than 1% difference in the predicted water depth above the weir crest.

Model output

Four different crest flows were simulated using CFD modelling. In addition it was assumed that at the very lowest heads over the crest the discharge coefficient would approximate to the broad crested weir value of 1.71. These five reference points in each case then enabled additional values to be derived by interpolation. The discharge coefficient curves, derived by curve fitting through the obtained points, are shown in Figure 4 for both Loyne and Cluanie. It can be seen that both crests exhibit discharge coefficient values greater than 2.00 as the heads on the crests increase.

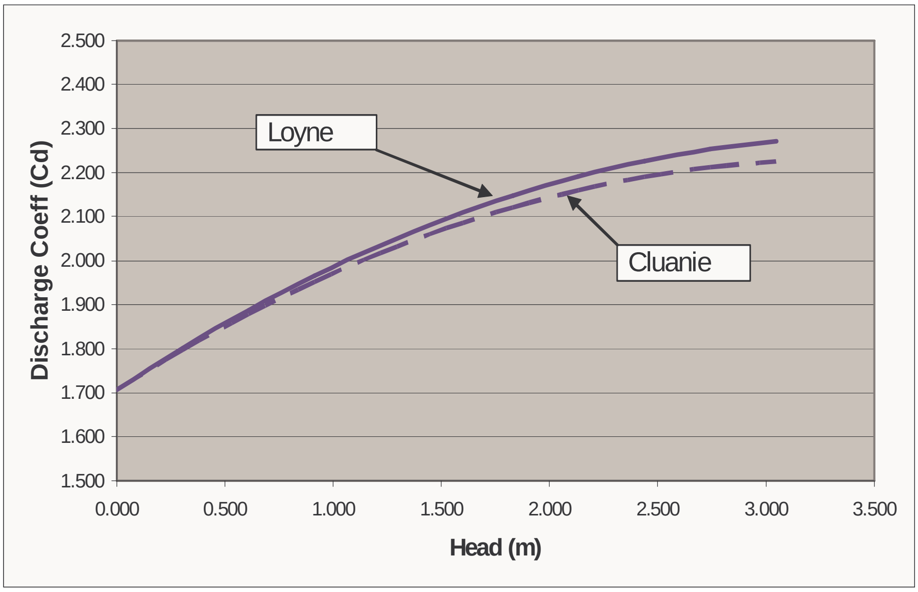


Figure 4. Derived discharge coefficients plotted against head

It should be noted that the suspected "design" heads for the Loyne and Cluanie crests were 4ft and 5 ft respectively and that for a standard ogee type crest profile and vertical upstream face the metric discharge coefficient at the design head will be in the order of 2.18. In fact the Loyne and Cluanie crests achieved 2.06 at these heads.

RESULTS OF FLOOD ROUTING USING THE REVISED DISCHARGE COEFFICIENTS

The previous PMF flood assessments had been carried out by others in 2004 using a non-variable crest discharge coefficient of 2.00 for Loyne and Cluanie dams and by routing PMF inflows in conjunction with snow-melt rates of both 1.75 mm/hr and 5.00 mm/hr. Both cases were re-run for Loyne and Cluanie dams using the variable discharge coefficients derived by CFD modelling. The results are shown in Table 2.

The general standard "base" cases are those featuring a snow-melt rate of 1.75 mm/hr. In those cases the use of a variable discharge coefficient produced a small, but useful, reduction in the maximum still-water reservoir level reached. In both cases it also, inevitably, produced a marginal increase in the maximum outflow, something which should be reflected in any downstream inundation studies.

Table 2. Derived Maximum Still-Water Reservoir Levels

Snow-melt (mm / hr)	Head over Crest (m)		Head reduction		Maximum outflow (m^3/s)	
	2004 studies	with variable Cd	(m)	%	2004 Studies	with variable Cd
Loyne						
1.75	1.82	1.77	0.05	2.7%	336	349
5.00	2.29	1.98	0.31	13.5%	475	421
Cluanie						
1.75	1.59	1.44	0.15	9.4%	426	458
5.00	2.15	1.58	0.57	26.5%	534	533

The additional cases featuring snow-melt rates of 5.00 mm/hr produced particularly useful reductions in head over the crests, however, these were partly due to hydrological assessment errors in the 2004 studies.

MORE READILY ACCESSIBLE ALTERNATIVE METHODS

Clearly it is advantageous to accurately model the discharge characteristics of weirs as part of any routine flood safety assessment. However, not all will have ready access to CFD modelling facilities and in most cases it will, anyway, be sufficient to use a, simplified and approximate method.

In the case of a standard ogee crest, discharge coefficients at the design ahead are readily available from sources such as "*Design of Small Dams*" published by the USBR. These are based on experimental and prototype data and are repeated in a number of specialist textbooks. Furthermore the same sources will indicate how the discharge coefficient will vary according to the slope on the upstream face of the weir crest and also according to how upstream levels vary above and below the design head (*Hd*).

For a standard ogee crest with a vertical upstream face a discharge coefficient of 2.18 is generally assumed at the design head. This will increase by approx 7% at 1.6 times *Hd* and reduce by 10% at 0.40 times *Hd*. Equations can be developed to describe this relationship. An approximate one in metric units, and covering the range from +/- 50% of *Hd* gives a value for Coefficient *C* of:-

$$C = Cd\ [\ 0.87 + 0.125\ (\ H / Hd)\ ]$$

Where

H = the head on the weir

C = the effective discharge coefficient at head H

Hd = the design head

Cd = the discharge coefficient at the design head

However it is also necessary to assess the original "Design Head" in order to use this. This can be approximated, as well as possible discharge coefficients, using publications like USBR Monograph 9 on "*Discharge Coefficients for Irregular Overfall Spillways*. Alternatively the basic equation for an ogee crest is:-

$$Y = X^{1.85} / (2 \, . \, Hd^{0.85})$$

Where

X = the horizontal distance downstream of the crest apex

Y = the vertical distance down from the crest apex

Taking a number of X,Y distance measurements on a given crest, from the crest apex, it should be possible to back-analyse a value for Hd which can then be used in subsequent flow equations. In the case of Loyne and Clunie it was possible to compare the ogee profiles in this way to standardised profiles and derive values for Hd of 4ft (1.219m) and 5ft (1.524m) respectively. However the upstream crest extensions at both dams led to doubts about how accurate the direct use of ogee based values would be and hence led to the use of CFD modelling. As discussed earlier, in fact a potential ogee crest discharge coefficient of 2.18 was reduced to 2.06 at these "design" heads, by the upstream extensions.

Lastly it should be noted that the derivation of equations is not always necessary in order to carry out flood routing checks. Modelling programmes such as Micro-FSR can route a weir outflow using tabulated values of discharge against head and interpolate between them to obtain any necessary intermediate values.

CONCLUSIONS

It can be seen that the use of variable weir coefficients not only reflects engineering reality but can lower the maximum reservoir levels obtained from routine flood safety assessments. However, such a reduction may also be accompanied by an increase in maximum outflow which should be reflected in any downstream inundation studies.

In the case of overspill crests with standard shapes, published data can be used to derive appropriate discharge characteristics as well as to assess how those characteristics will vary with head over the weir. In the case of non-standard crests, CFD modelling techniques can be used to make an assessment of the weir discharge characteristics much more rapidly and inexpensively than would be the case using a physical hydraulic model.

Although the benefit in terms of water level reduction for the standard PMF scenario may seem small, it may prove of significance in ongoing stability reviews for both structures. Should more extreme PMF (5 mm/hr snowmelt scenarios) be adopted then the benefits will be greater.